1. What is the difference between scientific computing and numerical analysis? Hint: Search on the internet may be good way to answer this questions.

Numerical analysis is developing methods and algorithms, for all of mathematics, especially getting them to be faster and more accurate. Scientific Computing is applying these algorithms to practical means, applying algorithms in the world of science to generate models. It seems like Numerical is the more theoretical side of these algorithms, scientific computing is actually using a computer and algorithms to find something.

1. Define both absolute error and relative error. Give an example where relative error is a better measure of error and an example where the absolute error is more appropriate.

Absolute error is how far off our approximation is from the real value. Relative error expresses how large the error is compared with the total size of the thing being measured. Or Relative error = Absolute error/Actual Value. If you were doing an error analysis The Relative error would be more useful. But if you were looking at just a single measurement it would be useful to have the relative error, since it gives you more context in the question. Also if you had really big measurements that needed to be very precise it should be better to have the absolute error, since there wouldn’t be a large change in relative error.

1. What are major differences between roundoff error and discretization error?

Roundoff error comes when numbers cannot be completely defined by a computer. An example of this would be any method that uses pi, since there are infinite decimal places pi any pi used will have to be rounded to a finite number of decimal places, this method would have some kind of roundoff error. Discretization error comes when computers represent continuous functions, are evaluated by a finite number of evaluations.

1. Assuming accumulation of roundoff error will happen, discuss what it means to mitigate against adverse effects on the approximation of solutions of mathematical problems.

Since we know that there will be roundoff error often no matter what we do, we try and mitigate the effect of the this error, or try and minimize how big this error will be. One way to this would be using the double precision floating point numbers instead of single precision floating point numbers since double precision will be much more accurate than single precision, and if we cut it off the round of error will be less.

1. Define the term flop in terms of computing. How can the concept of flops be used to assess the accuracy of any numerical algorithm.

Flops is an acronym for Floating Point Operations per second. I cannot find anything about flops related to accuracy of an algorithm, it is more of a measurement of efficiency and speed of the calculations. Maybe low flops are also associated with calculations with more error as they might not be as accurate since they take more time, and steps to find.

1. Distinguish between problem conditioning and algorithm stability. Clearly define both problem conditioning and algorithm stability.

Well conditioned problems will have small input changes lead to small output changes, ill conditioned problems are the opposite, small input changes lead to large output changes. Algorithm is very similar but it deals with algorithms and not singular statements. All it takes for an algorithm to become unstable is a single any step is ill-conditioned.

1. Define the term machine precision and explain the importance of this parameter in scientific computing.

Every number in a computer is defined by a finite number in base two. Computers represent decimals and large numbers as base 2 exponential. So, in every machine there is a limit to how small of a number can be used. The smallest number will cause most of the round off error, so for calculations that need to be very very accurate we need to take this machine precision into consideration.

1. Define underflow and overflow in floating point number systems.

Underflow is when the result of an operation is smaller than the smallest number that can be represented by the machines floating point numbers. Overflow is when a result of an operation is greater than the largest number that can be represented by the machine’s floating point number.

1. How does underflow impact computations in numerical algorithms?

Usually underflow errors can just be replaced by zero, as it is just a really close number to zero. We then just need to account for this error when we report numbers.

1. Define the term “catastrophic cancellation”. How does this effect computations.

When you subtract two approximations from each other, they may be very different from the difference of the actual numbers. Or the difference of two approximations, is very different from the difference of their original counterparts. It makes the difference between two approximations is not a good approximation of the difference of the originals.

1. Use an example like finite difference approximation of derivatives to discuss the impact of cancellation on approximations on computers.
2. Determine the machine precision for a 64-bit representation of any real number in the resulting computer number system.

This would be the pseudo code for a method that would find the machine precision on any machine.

epsilon = .5

X = 1

Loop while error > 0 {

error = |x + epsilon|

epsilon = epsilon/2

}

The resulting epsilon after this loop should be the machine precision of the given computer.

1. What is a nonlinear function? Define the root finding problem for an arbitrary nonlinear function

A nonlinear function is a function that cannot be defined using a linear equation, or mx +b = y. Roots are where these functions output is zero. So the root finding problem is finding the input values that output 0.

1. What theorem is used as a basis for the Bisection method for finding roots of nonlinear functions.

Intermediate value theorem, since bisection only works for sure If the root changes sign, if it does change sign in some interval there must be at least one root inside of that interval.

1. For the root finding problem, define an auxiliary fixed point equation and a fixed point iteration to generate a sequence of approximations for the root finding problem.

X0 – function

Loop

X1 = x0 – function

Error = abs(x1-x0)

X0 = x1

1. Use Taylor series approximations to determine the convergence of fixed point iterations for the root finding problem.
2. What is the downside of using fixed point iteration to find the roots of nonlinear functions?

Fixed point iteration only converges if the initial guess is really close to the actual root.

1. Newton’s method is the gold standard for finding roots of nonlinear functions. Why is this the case? Use comparisons to Bisection and fixed point iteration in your response.

It doesn’t need many iterations, and it is a generally the most accurate. So it runs fast and is very accurate, and it only needs one initial guess.

1. What restrictions exist on fixed point iteration for finding roots of nonlinear functions?

If initial value is not very close to the actual value of the root, the method will not converge to a root. It will only converge when |g’(x)|< 1

1. What restrictions exist on the Bisection method for finding roots of nonlinear functions?

Bisection only converges on a root if the root changes signs from one side to the other. It is also comparably slow, and may not be as accurate as other methods. It has a fixed rate of convergence thus it may need a larger number of iterations to find a root.

1. What restrictions exist on Newton’s method for finding roots of nonlinear functions?

It doesn’t always converge. It also requires one to find the derivative of the function, which might not exist, or be a lot more computationally intense then just using another root finding method.

1. Why would a person choose to use the Secant method over either the Bisection method or Newton’s method for finding roots of a nonlinear function.

If the function either has a complicated derivative, or doesn’t have a derivative, it should be easier to use the Secant Method. If the function’s root doesn’t have different signs on its sides bisection won’t work, but secant still will, and we can give it a similar interval.

1. What restrictions exist on the Secant method for finding roots of nonlinear functions.

Secant method doesn’t always converge when given intervals not to close enough to the root. It is also much slower than Newton’s method.

1. What are hybrid methods (Bisection plus Newton’s method) for finding roots of nonlinear functions?

Hybrid methods are methods that implement two different algorithms. In one case is an algorithm that uses bisection until it a point is reach where newton’s method will for sure converge. This is great because bisection will always converge if given a proper interval, but we get to use newton’s method which is more accurate but can only be used when given, a close root approximation.

1. Show that the Bisection method is a first order method using relationships between successive errors in the iteration formula.

It usually only works with functions that have roots that have a change of sign, It has a fixed rate of convergence, so it will need a lot more iterations to reach a given degree of precision.

1. What is git? How is git helpful in scientific computing?

It is a great way to stay organized and have many projects in the same place. It also makes it easy to work with other people. Or work on the same project on many different machines. If mistakes are made, git makes it really easy to revert files to older versions.

1. What is the difference between synchronous and asynchronous computations?

Synchronous computations in order. Asynchronous are done in almost any order. Asynchronous can utilize the true power of having multiple core CPUs.